$$xu'^2 + yv'^2 + tu'v' = f(\varepsilon) \tag{5.4}$$

where f is an arbitrary analytic function.

To obtain the corresponding solution of system (3.2) it is convenient to select $\varepsilon = v$ as the parameter. Changing to real variables by means of (5.1) we obtain the double wave of system (3.2)

$$\theta = \varphi(\omega_1, \omega_2), \quad \omega_3 = \psi(\omega_1, \omega_2)$$

in which the functions φ, ψ satisfy the Cauchy-Riemann conditions.

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THE GEOMETRICAL CHARACTERISTICS OF EQUALLY-STRONG BOUNDARIES OF ELASTIC BODIES*

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The necessary conditions for the existence of systems of surfaces or plane curves of special shape determined from mechanical considerations, by potential theory methods, are found, a number of integral identities is constructed, and certain modifications of the Robin problem are solved.

1. A linearly elastic homogeneous and istoropic three-dimensional domain S of the space E is considered which is weakened by a set of m non-intersecting closed cavities S_k^- with smooth boundaries Γ_k $(k = 1, 2, \ldots, m)$ and is loaded by remote forces P_i $(i = 1, 2, 3)$ along the axes of an $X_1X_2X_3$ Cartesian coordinate system, G, ν are the elastic moduli of the medium, and $I_1(x)$, $I_2(x)$ are stress tensor invariants at an arbitrary point $x = (x_1, x_2, x_3)$.

The boundary $\Gamma = \bigcup \Gamma_k$ is called equally-strong for a given load [I] if the identity $I_1(\xi) = \mathrm{const}$ holds at any of its points $\xi = (\xi_1, \xi_2, \xi_3)$. The constant on the right-hand side equals $P_1 + P_2 + P_3 = P$. It is proved in [I] that such a boundary minimizes the maximum value, over the domain, of the local Mises plasticity criterion $F(x) = I_1^2(x) - 3I_2(x)$, thereby being the solution of the following optimal control type problem:

$$\max_{x \in (S+\Gamma)} F(x) \to \min_{\{\Gamma\}} \tag{1.1}$$

Since the function $F(x)$ is invariant under a similarity transformation of the coordinates, the optimal boundary according to (1.1), if it exists, is not defined uniquely, but to at least the accuracy of a scale given by an arbitrary factor C. Indeed, the class of solutions is significantly broader in many cases, which is utilized substantially in Sect.3.

It has been established /1/ that the components of the displacement vector $\mathbf{u}(x)$ of the state of stress corresponding to a perturbation induced in the homogeneous field of cavities are harmonic functions in the domain S that decrease at infinity as $O(|x|^{-2})$, take values on the optimal boundary that are proportional to the corresponding coordinate at the point

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$\xi \in \Gamma_k$ /1/

$$u_i(\xi) = \lambda_i \xi_i + d_{ik}, \quad 4G\lambda_i = P - 2P_i$$
$$i = 1, 2, 3; \quad k = 1, 2, \ldots, m \tag{1.2}$$

where d_{ik} are certain constants, and moreover

$$\operatorname{div} \mathbf{u}(x) \equiv 0, \quad \operatorname{rot} \mathbf{u}(x) \equiv 0, \quad x \in S \tag{1.3}$$

In combination with the loading boundary conditions /2/

$$2G\partial u_i/\partial n = -P_i n_i, \quad i = 1, 2, 3 \tag{1.4}$$

the optimality relationships (1.2) form an inverse boundary value problem of elasticity theory that afford a constructive possibility for finding the shape of the equally-strong boundary /1, 3/ in a number of cases. Here $\mathbf{n} = (n_1, n_2, n_3)$ is the unit vector of the internal normal to Γ for S at the point ξ.

By using their individuality, the special form of the right-hand sides of the identities (1.2) and (1.4) enables us to construct harmonic continuations of the components of the vector $\mathbf{u}(x)$ within the domain $S_- = US_k^-$, respectively by the functions $\lambda_i x_i'$ or $-P_i x_i'$ $(i = 1, 2, 3)$, $x' = (x_1', x_2', x_3') \in S_-$. The vectors $\mathbf{U}_1$ and $\mathbf{V}_1$ obtained in this manner are defined in all space, have a given asymptotic form at infinity, are harmonic in S and S_-, but possess different properties at optimal boundary points. Namely, it follows from (1.4) that the vector $\mathbf{U}_1$ is continuous on Γ, but experiences a jump in the normal derivatives there

$$4G\mu_1(\xi) = [\partial \mathbf{U}_1/\partial n^+ - \partial \mathbf{U}_1/\partial n^-] = -P\mathbf{n} \tag{1.5}$$

On the other hand, it follows from (1.2) that the jump in $\mathbf{V}_1$ on Γ equals

$$4G\mu_2(\xi) = [\partial \mathbf{V}_1/\partial n^+ - \partial \mathbf{V}_1/\partial n^-] = P(\xi_1, \xi_2, \xi_3) \tag{1.6}$$

and its normal derivatives are continuous.

Relations (1.5) and (1.6) enable us /4/ to write $\mathbf{U}_1$ and $\mathbf{V}_1$ in terms of the integral operators Λ_1, Λ_2 of a simple or double layer, respectively, of the given densities

$$4\pi \mathbf{U}_1(x) = \Lambda_1[\mu_1(\xi)] = -P\Lambda_1[\mathbf{n}(\xi)]$$
$$4\pi \mathbf{V}_1(x) = \Lambda_2[\mu_2(\xi)] = -P\Lambda_2[\xi_1, \xi_2, \xi_3] \tag{1.7}$$

Differentiating the first of identities (1.7) with respect to n and then passing to the limit $x \to \eta \in \Gamma$ in all the relationships obtained we obtain in scalar form, taking (1.2) and (1.4) and the properties of the potentials into account /4/,

$$\eta_i + d_{ik} = \beta_i \Lambda_i [\partial \xi_i/\partial n]; \quad \beta_i = P/(2P - 4P_i) \tag{1.8}$$

$$\eta_i + d_{ik} = \gamma_i \Lambda_2 [\xi_i + d_{il}]; \quad \gamma_i = P/(P - 4P_i) \tag{1.9}$$

$$\partial \eta_i/\partial n = \gamma_i \Lambda_2^* [\partial \xi_i/\partial n] \tag{1.10}$$

$$\eta = (\eta_1, \eta_2, \eta_3) \in \Gamma_k, \quad \xi \in \Gamma_l; \quad k, l = 1, 2, \ldots, m$$

$$\beta_i = \gamma_i/(\gamma_i + 1), \quad i = 1, 2, 3 \tag{1.11}$$

(Λ_2^* is the operator conjugate to Λ_2).

It follows from the representations obtained that the constants d_{ik} therein are zero. Indeed, by taking into account that the operator Λ_2 of an arbitrary constant reduces to a Gauss integral /4/, and consequently, is calculated explicitly, relation (1.9) can be written in the form of inhomogeneous Fredholm-type integral equations

$$\eta_i - \gamma_i \Lambda_2(\xi_i) = (\gamma_i - 1) d_{ik} \tag{1.12}$$

The identity (1.10) means that $\partial n_i/\partial n$ is the eigenfunction of the operator Λ_2^* corresponding to the eigennumber γ_i $(i = 1, 2, 3)$. Since all the points of the spectrum of the operators under consideration are simple /4/, this function is unique. It is obviously orthogonal to all d_{ik}; consequently, each of the Eqs. (1.12) has a unique solution /4/. It is verified directly that it reduces to the constant $(-d_{ik})$ on the surface Γ_k but this contradicts (1.12), and, consequently, $d_{ik} = 0$.

Therefore, η_i is the eigenfunction of the operator Λ_2 for the number γ_i, with support on the equally-strong boundary, while the identity (1.8) resulting from (1.9) and (1.10) is, together with (1.11), a special case of the relationships obtained /5/ with respect to any eigenfunctions of the mutually conjugate operators Λ_2, Λ_2^* on arbitrary smooth surfaces. It should be noted that the proof presented in /5/ is not accurate and needs obvious revisions.

The spectrum of integral operators of potential type lies outside the unit circle, as is well-known /4/; consequently $|\beta_i| > 1$ $(i = 1, 2, 3)$, which is equivalent to the triangle inequality

$$|P_1| \leqslant |P_2| + |P_3| \tag{1.13}$$

and two analogous ones obtained from it by cyclic permutation of the subscripts. According to the above, they are the necessary condition for the solvability of problem (1.1) in this formulation. For a single cavity, these inequalities are also sufficient /1/, a triaxial ellipsoid surface with a ratio between the axes dependent on the load is optimal.

Relations (1.8)-(1.11) are obtained as a result of clear simplification of the elastic equilibrium equations of the medium in domains with equally-strong boundaries. They are also valid in the two-dimensional optimization problem for a plane with equally-strong holes for $\gamma_i = P(P-2P_i)$ $(i = 1, 2)$, $P = P_1 + P_2$. The condition for its solvability takes the form

$$q = |(P_2 - P_1)/(P_1 + P_2)| \leqslant 1 \tag{1.14}$$

Earlier /6/, the univalency of the conformal mapping of a domain with unknown boundaries into a standard domain while actually seeking them was posed as a requirement.

The identities mentioned describe the geometry of a set of equally-strong surfaces defined in problem (1.1) and parametrically dependent on values of the load P_i. The latter can vary within limits which are not wider than the allowed inequalities (1.13).

2. In the case $m > 1$ the components $U_{1i}(x)$ of the vector $\mathbf{U}_1$ are an extension of first-order ellipsoidal harmonic functions /2/ since they allow of the integral representation (1.7) and agree in S_- with the linear harmonic polynomials $\lambda_i x_i$. By using them, three (out of a possible five) analogous second-order functions can indeed be constructed by means of the formula

$$\mathbf{U}_2(x) = (U_{21}, U_{22}, U_{23}) = \mathbf{R}(x) \times \mathbf{U}_1(x),\ x \in E$$

$(\mathbf{R}(x) = (x_1, x_2, x_3)$ is the radius-vector of the point x).

It follows from (1.3) that the vector $\mathbf{U}_2(x)$ is also continuous everywhere and harmonic in S and S_- (∇^2 is the Laplace operator):

$$x \in S,\quad \nabla^2 \mathbf{U}_2(x) = \operatorname{rot} \mathbf{u}(x) \equiv 0$$
$$x \in S_-,\quad \nabla^2 U_{2i}(x) = 2(P_l - P_j) x_l x_j \equiv 0$$
$$i \neq j \neq l,\ i, j,\quad l = 1, 2, 3$$

Taking account of (1.5) we conclude that the jump in the normal derivative of the vector $\mathbf{U}_2$ on Γ equals

$$\mathbf{R}(\xi) \times [\partial \mathbf{U}_1/\partial n] = -P\mathbf{R}(\xi) \times \mathbf{n}(\xi)$$

from which a representation of the type (1.7) results

$$\mathbf{U}_2(x) = -P\Lambda_1[\mathbf{R}(\xi) \times \mathbf{n}(\xi)] \tag{2.1}$$

These functions are sufficient to construct a closed solution in S for the so-called Robin elastostatic problem of the second kind /2/. It consists of determining the state of stress and strain of an elastic medium that occurs during rotation through (small) angles θ_i around the axes X_i of a system of absolutely solid bodies included therein that occupy the domain S_-.

It turns out that in the case of equally-strong boundaries the solution is independent explicitly of m and is constructed according to a scheme presented in /2/ for a single inclusion in the shape of a triaxial ellipsoid. To this end, the appropriate displacement vector $\mathbf{V}(x)$ is sought in the Papkovich-Neuber form /2/ ($\mathbf{B}(x)$ is a vector and $B_0(x)$ is a scalar)

$$\mathbf{U}(x) = 4(1-\nu)\mathbf{B} - \operatorname{grad}(\mathbf{R}\cdot\mathbf{B} + B_0) \tag{2.2}$$
$$B_0 = N_1\theta_1 U_{21} + N_2\theta_2 U_{22} + N_3\theta_3 U_{23},\quad B_1 = D_1\theta_2 U_{13} - D_1'\theta_3 U_{12}$$

The components B_2, B_3 are obtained from B_1 by cyclic permutation of the subscripts. Substitution of (2.2) into the boundary condition of the problem /2/ $V_1(\xi) = \theta_2\xi_3 - \theta_3\xi_2$ and two analogous to it also result in a linear system of algebraic equations in the unknown constants N_i, D_i, D_i' which separate, as in /2/, into three that correspond separately to rotations through the angles θ_i. Thus, for θ_1 the system agrees, apart from the notation, with (5.4.5) of /2/ if the optimality relationships [I] for an ellipsoid are taken into account. It is not given here to conserve space.

The partial derivatives of the components $\mathbf{U}_1, \mathbf{U}_2$ with respect to x_i on Γ needed to construct the system are found from (1.7) and (2.1) by the Hugoniot-Hadamard differentiation formulas /4/. For instance

$$\frac{\partial U_{11}}{\partial x_1} = \left[\frac{\partial U_{11}}{\partial n}\right] n_1 + \frac{\partial U_{11}}{\partial x_1^-} = -Pn_1^2 + \lambda_1$$
$$\frac{\partial U_{21}}{\partial x_1} = \left[\frac{\partial U_{21}}{\partial n}\right] n_2 + \frac{\partial U_{21}}{\partial x_1^-} = Pn_2(n_3\xi_2 - n_2\xi_3)$$

3. We will now consider the two-dimensional problem (1.1) $(P_3 = 0,\ P = P_1 + P_2)$. For a large number of different modifications of the hole arrangement in the plane their optimal boundary is found explicitly /6/ as a dependence of the form

$$\xi_2 = f(\xi_1, m, q, C, \omega_1, \omega_2, \ldots, \omega_{2m}) \tag{3.1}$$

where f is a definite function for each m and $\omega_k\ (k = 1, 2, \ldots, 2m)$ are geometric parameters that are coordinates of the ends of m slits along the real axis of the auxiliary plane on which S is mapped in /6/ while seeking Γ from conditions (1.2)-(1.4). Under additional symmetry their number can be reduced.

It is important that inequality (1.14) is not only a necessary but a sufficient condition in all the cases considered in /6/, that ensure the existence of equally-strong boundaries for any values of ω_k with the natural constraint

$$-\infty < \omega_1 < \omega_2 < \ldots < \omega_{2m} < \infty$$

denoting that the slits on the auxiliary plane do not intersect. Therefore, (3.1) exhaustively describes a $2m$ parametric family of solutions of the plane problem (1.1) for a given q.

On this basis, the ordinary Robin problem, that consists /4/ of determining the density $\rho(\xi)$ of a logarithmic simple layer potential with support on Γ that takes constant values in S_k^-, can be solved analytically, which is equivalent to constructing a multiconnected analogue of the zero-order ellipsoidal harmonic function $U_0(x)$ in E (t is the arclength of the contour Γ):

$$U_0(x) = \int\limits_\Gamma \rho(\xi) \ln \frac{1}{r(\xi, x)}\, dt, \quad r = |x - \xi|,\ \xi \in \Gamma \tag{3.2}$$
$$U_0(x') = \text{const}, \quad x' \in S_-; \quad \nabla^2 U_0(x) \equiv 0, \quad x \notin \Gamma$$

Although only m linearly independent solutions of this problem /4/ exist in a broad class of domains of different geometry, the function $\rho(\xi)$, insofar as is known, is actually found only for an ellipse /2/ as the boundary of one optimal hole /6/

$$\rho(\xi) = D/H_\nu(\mu, \nu_0) = D\sqrt{1 + \nu_0^2}/\sqrt{1 - \mu^2} \tag{3.3}$$
$$\xi_1 = C\mu\nu_0, \quad \xi_2 = C\sqrt{(1 - \mu^2)(1 + \nu_0^2)}, \quad |\mu| \leqslant 1$$
$$\xi_1^2/\nu_0^2 + \xi_2^2/(1 + \nu_0^2) = C^2$$

where μ, ν_0 are elliptical coordinates of the point ξ on the ellipse $\nu = \nu_0$, H_ν is the Lamé coefficient therein, and C and D are constants.

Let us consider the general case $m \geqslant 1$. It follows /1/ from (1.3) that equally-strong boundaries possess a characteristic property: the internal gravitation potential of the masses distributed uniformly within them is a given quadratic form of the coordinates

$$\varphi(x) = \frac{P}{2\pi} \int\limits_{S_-} \ln \frac{1}{r(x, x')}\, dx' = b_k^{(m)} - P_2 x_1^2 - P_1 x_2^2 \tag{3.4}$$
$$x' \in S_-, \quad x \in S_k^-$$

We now assign an optimal boundary Γ with parameters $\{\omega_j\}$ and without changing q we select a new $\{\omega_j'\}$ such that each contour Γ_k lies strictly within the corresponding Γ_k' (Fig.1). According to the above, there exist infinitely many such pairs Γ and Γ'. According to (3.4), the gravitational potential obtained by this method for a system of annular domains retains a constant value within $(S_k^- + \Gamma_k)$

$$\varphi(x) = \frac{P}{2\pi} \int\limits_{S_-' - S_-} \ln \frac{1}{r(x, x')}\, dx' = \frac{P}{2\pi} \int\limits_{S_-'} \ln \frac{1}{r(x, x')}\, dx' - \tag{3.5}$$
$$\frac{P}{2\pi} \int\limits_{S_-} \ln \frac{1}{r(x, x')}\, dx' = \Delta b_k^{(m)}, \quad x \in S_k^- + \Gamma_k, \quad k = 1, 2, \ldots, m$$

which generalizes the two-dimensional analogue of Newton's theorem /7/ on the absence of attraction within a constant density elliptical ring to the multiconnected case.

We prove that a closed expression can be constructed for the density $\rho(\xi)$ for an arbitrary system of equally-strong contours in terms of the function f from (3.1) by a passage to the limit in the identity (3.5).

Let the boundary Γ' described earlier be obtained from Γ by a small variation of the form $(\delta\xi_2, \delta\xi_1) = hn$. The parameter $h(\xi)$ is the thickness of the ring between Γ_k' and Γ_k

evaluated along $\mathbf{n}$; consequently

$$h(\xi) = \sqrt{(\delta\xi_1)^2 + (\delta\xi_2)^2} = (\sqrt{1 + f_\xi^2}/f_\xi)\delta\xi_1 \qquad (3.6)$$

$$f_\xi = \partial f/\partial\xi_1$$

since the quantities $\delta\xi_1$, $\delta\xi_2$ are connected by the equation of the normal

$$f_\xi\delta\xi_2 + \delta\xi_1 = 0 \qquad (3.7)$$

For simplicity we require definite symmetry from the domain so that Γ from the other side goes over into Γ' by variation of a single parameter $\omega_1 = \omega$, say, on which we shall note the dependence later. Then we have

$$\delta\xi_2 = \delta f(\xi_1, \omega) = f_\xi\delta\xi_1 + f_\omega\delta\omega, \quad f_\omega = \partial f/\partial\omega$$

and expressing $\delta\xi_2$ from (3.7), we obtain

$$(1 + f_\xi^2)\delta\xi_1 = -f_\xi f_\omega\delta\omega$$

from which it follows that

$$h(\xi) = w(\xi)\delta\omega, \quad w(\xi) = -f_\omega/\sqrt{1 + f_\xi^2} \qquad (3.8)$$

The variation of the potential (3.4) due to the boundary motion is written in the form /8/

$$\delta\varphi(x) = \frac{P}{2\pi}\int_\Gamma w(\xi)\ln\frac{1}{r(x,\xi)}\,dt, \quad x \in S_- + \Gamma, \quad \xi \in \Gamma$$

According to (3.5) $\delta\varphi = \delta b_k^{(m)}(\omega)$, and therefore finally

$$\frac{P}{2\pi}\int_\Gamma w(\xi)\ln\frac{1}{r(\eta,\xi)}\,dt = \frac{\partial b_k^{(m)}(\omega)}{\partial\omega} \qquad (3.9)$$

$$\eta \in S_k^- + \Gamma_k, \quad k = 1, 2, \ldots, m$$

We conclude from a comparison of (3.9) with (3.2) that the function $w(\xi)$ is one of the solutions of the Robin problem. Here

$$\partial U_0(\xi)/\partial n = -(2\pi)^{-1}Pw(\xi)$$

The total mass $Q(\omega, q)$ of this neutral layer is determined just as simply. We divide each closed contour Γ_k into two parts by a line parallel to the X_1 axis such that the function $\xi_2(\xi_1)$ is defined uniquely from (3.1) in each of them. Let $y_1(\xi_1)$ and $y_2(\xi_1)$ be its corresponding branches and $\xi_{k1}(\omega)$ and $\xi_{k2}(\omega)$ partition points of Γ_k (Fig.1). Then taking (3.8) into account

$$Q = \int_\Gamma \rho(\xi)\,dt = \int_\Gamma f_\omega\,d\xi_1 = \sum_{k=1}^{m}\int_{\xi_{k2}(\omega)}^{\xi_{k1}(\omega)}\left(\frac{\partial y_1}{\partial\omega} + \frac{\partial y_2}{\partial\omega}\right)d\xi_1 = \qquad (3.10)$$

$$\frac{\partial A(\omega)}{\partial\omega}\sum_{k=1}^{m} y_1(\xi_{k1}) + y_2(\xi_{k1}) - y_1(\xi_{k2}) - y_2(\xi_{k2})$$

(A is the total area of all the holes). Here the rules for the differentiation of a definite integral with respect to a parameter are used /9/. In the majority of cases (3.10) simplifies since the sum vanishes because of the symmetry of S along the X_2 axis.

For instance, when $m = 1$, substitution of the equation of an ellipse from (3.3) into (3.8) results, as is required, in the first of relations (3.3). Here $A = \pi C^2\omega(1 - q^2)$ and $Q = 2\pi C^2\omega(1 - q^2)$. Less elementary cases are also examined analogously.

Two symmetric holes on the X_1 axis. The appropriate function $f(\xi_1, C, q, \omega)$ and the expression for the total area of the holes have the form /6/

$$|\xi_2(\xi_1)| = C(1 + q)[\mathrm{E}(\omega)\mathrm{K}^{-1}(\omega)F(\Psi, \omega)] - E(\Psi, \omega)$$

$$\Psi = \arcsin\left[\frac{C^2(1 - q^2) - \xi_1^2}{C^2(1 - \omega^2)(1 - q^2)}\right]^{1/2}$$

$$C\omega(1 - q) \leqslant \xi_1 \leqslant C(1 - q); \quad 0 \leqslant \omega \leqslant 1$$

$$A = 4\pi C^2(1 - q^2)[2 - \omega^2 - \mathrm{E}(\omega)\mathrm{K}^{-1}(\omega)]$$

(F, E are elliptic and K, E the complete elliptic integrals of the first and second kinds). Substitution of these expressions into (3.8) and (3.10) when taking account of formulas for the derivatives of elliptic integrals with respect to the parameter /10/ yields

$$\rho(\xi)=\frac{(1+q)\left[\mathbf{E}(\omega)\mathbf{K}(\omega)(1-\omega^2)\omega\sin 2\Psi+2B(\Psi,\omega)\right]}{2\left\{C^2\Delta(\Psi,\omega)+(1+q^2)(\xi_1{}^2-\mathbf{E}(\omega)\mathbf{K}^{-1}(\omega))^2\right]^{1/2}}$$

$$B(\Psi,\omega)=[\mathbf{E}(\omega)F(\Psi,\omega)-\mathbf{K}(\omega)E(\Psi,\omega)][\mathbf{E}(\omega)-\mathrm{K}(\omega)(1-\omega^2)]\,\Delta(\Psi,\omega);\quad \Delta(\Psi,\omega)=(1-\omega^2\sin\Psi)^{1/2}$$

$$Q=4\pi C^2(1-q)[2-2\omega-(2\mathrm{E}(\omega)-\mathrm{K}(\omega))(1-\omega^2-\mathrm{E}^2(\omega))\,\omega^{-1}(1-\omega^2)^{-1}]$$

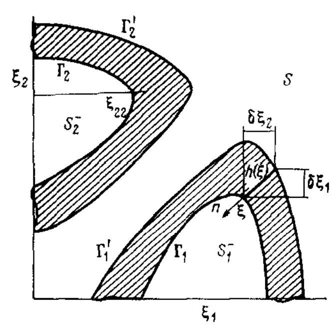

Fig.1

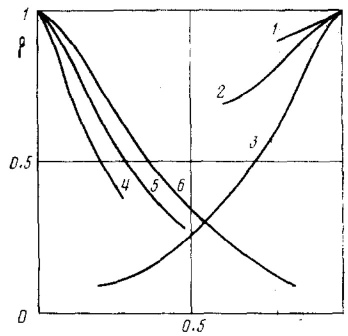

Fig.2

Fig.2 shows graphs of the function $\rho(\xi_1)$ normalized by the condition $\rho(1)=1$ for $q=0$ and $\omega=0.2;\ 0.4;\ 0{,}8$ (curves *1-3*).

A periodic system of holes arranged with a constant spacing h_0 on the X_1 axis. The shape of Γ is also found in /6/

$$|\xi_2(\xi_1)|=C(1+q)\ln\left|\frac{\cos\zeta}{\cos\omega}+\sqrt{\frac{\cos^2\zeta}{\cos^2\omega}-1}\right|\qquad h_0=2\pi C,\quad \zeta=C^{-1}(1-q)^{-1}x;\quad |\zeta|\leqslant\omega,\quad 0\leqslant\omega\leqslant\pi/2\qquad(3.11)$$

As compared with /6/, inaccuracies of a formal nature are corrected in (3.11). It is interesting that h_0 is independent of the load parameters P_1, P_2. It follows that (3.11) that

$$\rho(\xi_1)=\frac{C(1+q)\,\mathrm{tg}\,\omega\cos\xi_1}{[\cos^2\xi_1-\cos^2\omega+C(1+q)(1-q)^{-1}\sin^2\xi_1]^{1/2}}$$

$$Q(\omega)=\frac{\partial A(\omega)}{\partial\omega}=4C(1-q)\frac{\partial}{\partial\omega}\int_0^\omega\xi_2(\zeta)\,d\zeta=4C(1-q)\left[\xi_2(\omega)+\int_0^\omega\frac{\partial}{\partial\omega}\xi_2(\zeta)\,d\zeta\right]=8C^2(1-q^2)(\pi-2\omega)\,\mathrm{tg}\,\omega$$

Curves *4-6* in Fig.2 are graphs of $\rho(\xi_1)$ normalized by the condition $\rho(0)=1$ for $q=0.4$ and $\omega=0.3;\ 0.5;\ 0.9$.

In conclusion, we note that the constant value U itself of the function $U_0(\xi)$ on Γ and in S_- can be found by using the electrostatic analogy of the Robin problem /7/, when U is the charge distributed in an equilibrium manner over Γ, and U is the potential of the system of contours so that $C_0U=Q(\omega)$, where C_0 is the capacitance of Γ. As we know, it is conformally invariant and consequently, can be evaluated not for Γ but for the generating system of slits on the auxiliary plane mentioned at the beginning of Sect.3. For them C_0 is always expressed in quadratures /11/. The appropriate formulas are nore presented because of their length.

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